COMPARATIVE ANALYSIS OF HEAT TRANSFER ON FIN SURFACE WITH PARTIAL DIFFERENTIAL EQUATIONS

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Abstract: *Group classification of differential equations is an important method of numerical simulations. Present work focused on surface heat transfer on heat exchanger fins by numerical formulations, least square methods and collocation method used to compare the results in simulation. Non linear equations of heat transfer coefficient and surface temperature equations are solved. The tests for the dimensional-less temperature distribution and fin efficacy of various values of the problem parameters, obtained for comparison by means of these methods, are described relative to the methods adopted for viability and new. In addition, convergence problems for solving non-linear equations were found in the approach suggested to be very reliable, effective and practical for the problem discussed.*
Key words: *Numerical equations, heat transfer formulations, LSM, CM*

1.0 Introduction

Heat transport fins provide ventilation, oil-carrying pipes ventilation, cooling and electrical transformers, computer processor cooling, and air-conditioning, for example. Kern and Krause are providing an description of the expanded areas and their manufacturing applications[1]. A symmetrie analysis has been used to evaluate group classifications for the differential equation of the fin[2-3]. In another analysis the non-linear end equations of total temperature-dependent thermal conductance is studied by Pakdemirli and Sahin [4]. The exact analytical solution is possible in a basic state that continuously has thermal conductivity and heat transfer coefficients. Yet if there is a significant differential temperature in the fin, the coefficient of heat transfer and thermal conduction are not constant. As a consequence, typically, temperature properties are thermal conductivity and heat transfer coefficient.

As the precision of solving these non-linear problems is challenging, scientists concentrated on seeking various solutions to these types of equation, using semicalytical approaches, such as the PM [5], HPM [6–7], VIM, Homotopy Analysis (HAM) [10], Differential Process Transformation (DTM) [8-9], HAM, VIM [8-9]. Aziz and Hug and Aziz and Benzies [12] performed the oldest works on this subject. They solved the equation of the heat flow for a convective fin using a destructive approach to obtain linear temperature-dependent thermal conductivity. In this study, a nonlinear end with a thermal conductivity and thermal transfer coefficient depending on the power-law temperature is called.

Stern and Rasmussen[13] used the Colloquation Method (CM) to solve a linear difference equation of the third order. The possibility of using a technique of orthogonal collocation to overcome diffusiveness equation in the radial transient flow scheme was investigated by Vaferi et al.[14]. Recent collocation methods for the study of heat transfer through porous fins were used by Hatami et al. [15].Aziz and Bouaziz[16] are using the Least Square Approach (LSM) to determine how longitudinal fins should be handled. They find that the least quadratic form is simple relative to other methods of analysis. Shaoqin and Huoyuan[17], also Hatami et al.[18], Hatami and Ganji [19], Hatami and Domairry [20-21] and Ahmadi et al. [22] have established and analyzed Least-squares calculations for magnetohydrodynamic equations, as well as Hatami et al.[19].

2.0 Heat Transfer Equation

Some researchers found the thermal transfer equation of a fin with linear thermal conductivity depending on the temperature and power-law temperature dependent thermal transfer coefficient. It is a differential equation and limits in the following form.

$$(1+\beta\cdot\theta(x))\frac{d^2\theta(x)}{dx^2}-M^2(\theta(x))^{n+1}+\beta\left(\frac{d\theta(x)}{dx}\right)^2=0$$

With the following boundary conditions:

$$\theta'(0)=0,\ \theta(1)=1$$

2.1 Collocation Method (CM)

Assume that a differential operator D that operates on a function u in order to produce a function p [1]:

$$D(u(x))=p(x)$$

Function u can be approximated by a function called the function è a linear combination of simple functions chosen by:

$$u\cong\tilde{u}=\sum_{i=1}^{n}C_i\varphi_i$$

Residual as

$$E(x)=R(x)=D(\tilde{u}(x))-p(x)\neq 0$$

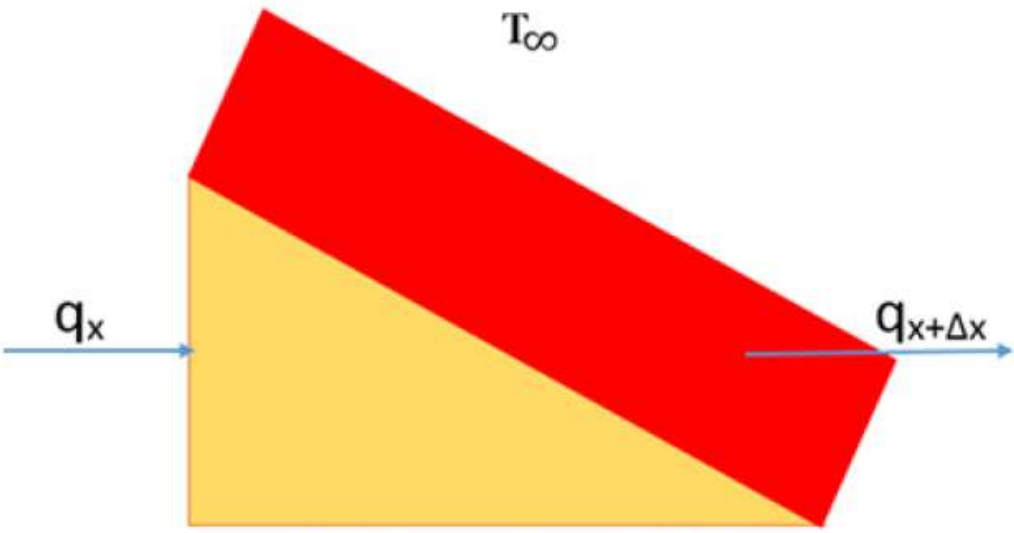


Fig. 1. Geometry of the problem

The idea is that the residual will be pulled over the domain through any way in the collocation. That is

$$\int_x R(x)\, W_i(x) = 0, \; i = 1, 2, \ldots, n$$

2.2 Least Square Method (LSM)

The Last Square Method of solving differential equations was an approximation method. Assume that a differential operator D is used to execute the function p:

$$D(u(x)) = p(x)$$

U is approximated by a U-turn function, which is a linear combination of basic functions chosen from an independent linear set. That is:

$$u \cong \tilde{u} = \sum_{i=1}^{n} c_i \varphi_i$$

The product of operations is usually p(x) when replaced by the differential operator, D. There would always be an defect or a residual:

$$R(x) = D(\tilde{u}(x)) - p(x) \neq 0$$

The notion in LSM is to force the residual to zero in some average sense over the domain. That is:

$$\int_x R(x) W_i(x) = 0 i = 1, 2, \ldots, n$$

If the continuous summation of all the squared residuals is minimized, the rationale behind the LSM’s name can be seen. In other words, a minimum of:

$$S = \int_x R(x) R(x) dx = \int_x R^2(x) dx$$

In order to achieve a minimum of this scalar function, the derivatives of *S* with respect to all the unknown parameters must be zero. That is:

$$\frac{\partial S}{\partial c_i} = 2 \int_x R(x) \frac{\partial R}{\partial c_i} dx = 0$$

Weight functions are seen to be:

$$W_i = 2 \frac{\partial R}{\partial c_i}$$

The "2" coefficient, though, can be higher, as the equation cancels out. Therefore, for the Least Squares approach the weight function is just the derivatives of the residual relative to the undefined constants:

$$W_i = \frac{\partial R}{\partial c_i}$$

3.0 Solving the differential equation with CM

Where the weight of Wi is precisely equal to the estimated weight of Ci constants in ieu. The tests for these unknown Cigarettes are n algebraic equations. The weighting functions are taken from the collocation process
the family of Dirac δ functions in the domain (i.e. $Wi(x) = \delta(x - xi)$). The Dirac δ function is defined as:

$$\delta(x - x_i) = \begin{cases} 1 & if\, x = x_i \\ 0 & Otherwise \end{cases}$$

The main aim of this method is to force the residual function becomes equal to zero.
Consider the trial function as:

$$\theta(x) = 1 + c_1\left(1 - x^2\right) + c_2\left(1 - x^3\right) + c_3\left(1 - x^4\right) + c_4\left(1 - x^5\right)$$

satisfies the boundary condition mentioned in Eq. (2). Substituting θ (*x*) into Eq. (1) leads to the residual function,
$R(c1;c2;c3;c4;x)$ as follow:

$$\begin{aligned}
&R(c_1,c_2,c_3,c_4,x) = \\
&45\beta x^8 c_4^2 + 72\beta x^7 c_3 c_4 + \left(56\beta c_2 c_4 + 28\beta c_3^2\right)x^6 \\
&+\left(M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_4 + 42\beta c_1 c_4 + 42\beta c_2 c_3\right)x^5 \\
&+\left(M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_3 + 30\beta c_1 c_3 + 15\beta c_2^2\right)x^4 \\
&+\begin{pmatrix} M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_2 \\ +20\beta c_1 c_2 - 20\beta c_1 c_4 - 20\beta c_2 c_4 - 20\beta c_3 c_4 - 20\beta c_4^2 - 20\beta c_4 - 20c_4 \end{pmatrix}x^3 \\
&+\begin{pmatrix} M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_1 \\ +6\beta c_1^2 - 12\beta c_1 c_3 - 12\beta c_2 c_3 - 12\beta c_3^2 - 12\beta c_3 c_4 - 12\beta c_3 - 12c_3 \end{pmatrix}x^2 \\
&+\left(-6\beta c_1 c_2 - 6\beta c_2^2 - 6\beta c_2 c_3 - 6\beta c_2 c_4 - 6\beta c_2 - 6c_2\right)x \\
&-M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_1 \\
&-M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_2 \\
&-M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_3 \\
&-M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_4 \\
&-M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n \\
&-2\beta c_1^2 - 2\beta c_1 c_2 - 2\beta c_1 c_3 - 2\beta c_1 c_4 - 2\beta c_1 - 2c_1 = 0
\end{aligned}$$

The residual function must be close to zero. For reaching this importance, four specific points in the domain
$x \in [0;1]$should be chosen as:

$$x_1 = \frac{1}{5},\quad x_2 = \frac{2}{5},\quad x_3 = \frac{3}{5},\quad x_4 = \frac{4}{5}$$

Eventually, a set of four equations and seven unknown coefficients can be obtained by substitutions to the residual function R(cn;x).

For example, for the specific case of n = 2 the approximate solution of the third order; M = 5; β = 2 is obtained, for which constants are determined (c1;c2;c3;c4) as follows:

***C*1 = −0*:*3034072316*; C*2 = −0*:*3877945372*; C*3 = 0*:*4909619407*; C*4 = −0*:*4074813190**

Those unknown parameters (C1;C2;C3;C4) which were reached by collocation method to a testing function would then easily obtain the reaction to the equat

$$\theta(x) = 0.3922788529 + 0.3034072316x^2 + 0.3877945372x^3 - 0.4909619407x^4 + 0.4074813190x^5$$

4.0 Solving with LSM

Because trial function must satisfy the boundary conditions in Eq. (2), so it will be considered as:

$$\theta(x) = 1 + c_1\left(1-x^2\right) + c_2\left(1-x^3\right) + c_3\left(1-x^4\right) + c_4\left(1-x^5\right)$$

Residual function will be as:

$$\begin{aligned}
R(x) &= 45\beta x^8 c_4^2 + 72\beta x^7 c_3 c_4 + \left(56\beta c_2 c_4 + 28\beta c_3^2\right)x^6 \\
&+ \left(M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_4 + 42\beta c_1 c_4 + 42\beta c_2 c_3\right)x^5 \\
&+ \left(M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_3 + 30\beta c_1 c_3 + 15\beta c_2^2\right)x^4 \\
&+ \begin{pmatrix} M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_2 \\ +20\beta c_1 c_2 - 20\beta c_1 c_4 - 20\beta c_2 c_4 - 20\beta c_3 c_4 - 20\beta c_4^2 - 20\beta c_4 - 20c_4 \end{pmatrix} x^3 \\
&+ \begin{pmatrix} M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_1 \\ +6\beta c_1^2 - 12\beta c_1 c_3 - 12\beta c_2 c_3 - 12\beta c_3^2 - 12\beta c_3 c_4 - 12\beta c_3 - 12c_3 \end{pmatrix} x^2 \\
&+ \left(-6\beta c_1 c_2 - 6\beta c_2^2 - 6\beta c_2 c_3 - 6\beta c_2 c_4 - 6\beta c_2 - 6c_2\right)x \\
&- M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_1 \\
&- M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_2 \\
&- M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_3 \\
&- M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n c_4 \\
&- M^2\left(-x^5c_4 - x^4c_3 - x^3c_2 - x^2c_1 + c_1 + c_2 + c_3 + c_4 + 1\right)^n \\
&- 2\beta c_1^2 - 2\beta c_1 c_2 - 2\beta c_1 c_3 - 2\beta c_1 c_4 - 2\beta c_1 - 2c_1 = 0
\end{aligned}$$

The R(x) function will be replaced in weight by a series of four equations and coefficients c1−c4 are calculated by solving this system of equations. Using Least Square Approach for example to fix this issue

when $n = 2; M = 5; \beta = 2$,θ (*x*) is as follows:

$$\theta(x) = 0.3863431577 + 0.3085306360x^2 + 0.5301979592x^3 - 0.8043042649x^4 + 0.5792325120x^5$$

5.0 Comparative results obtained after simulation

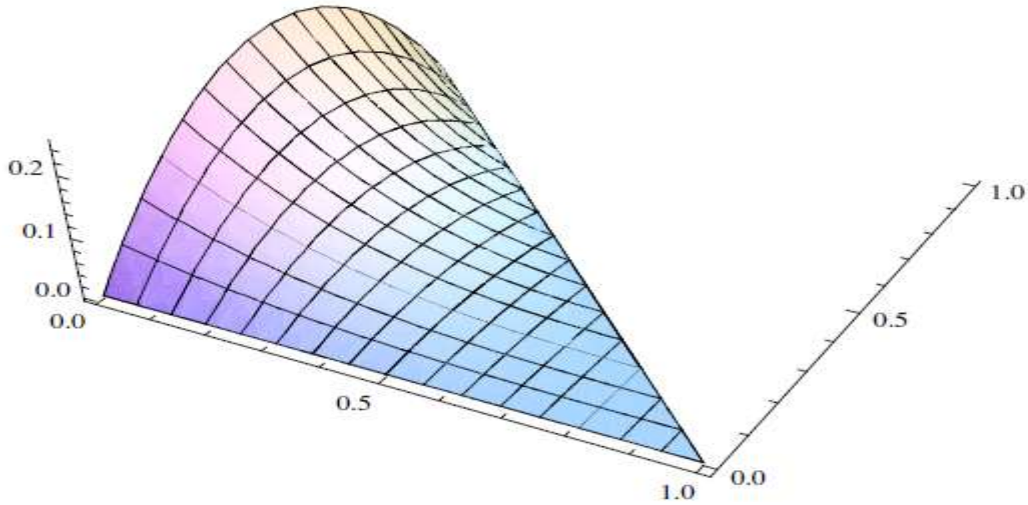
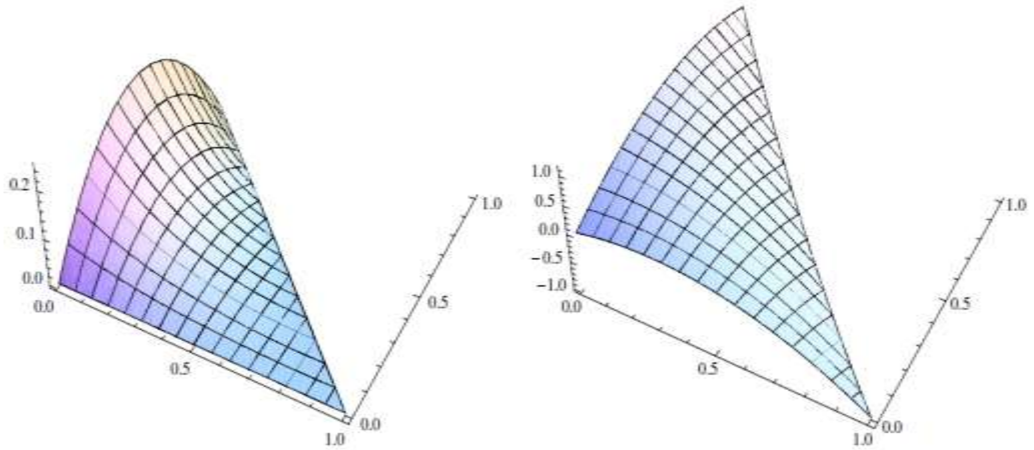
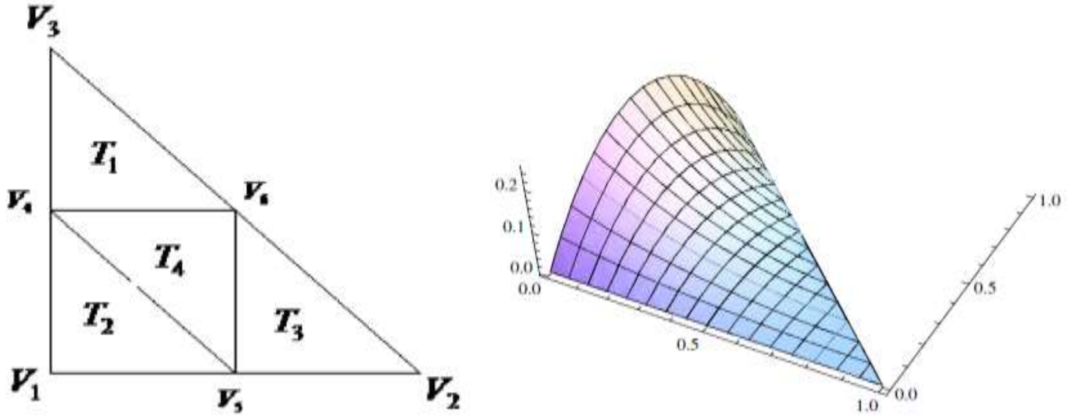


Figure shows exact solution after LSM method



Left: the approximate solution u(x; y); right: the residual function R(x; y).

Such criteria are added to the problem of minimization along with boundary conditions in order to create a specific question of optimisation. The solution for the optimization problem is the Bézier grade surface patch with Ck continuity.



Left: subdivision; right: the approximate cubic spline surface u= τ.

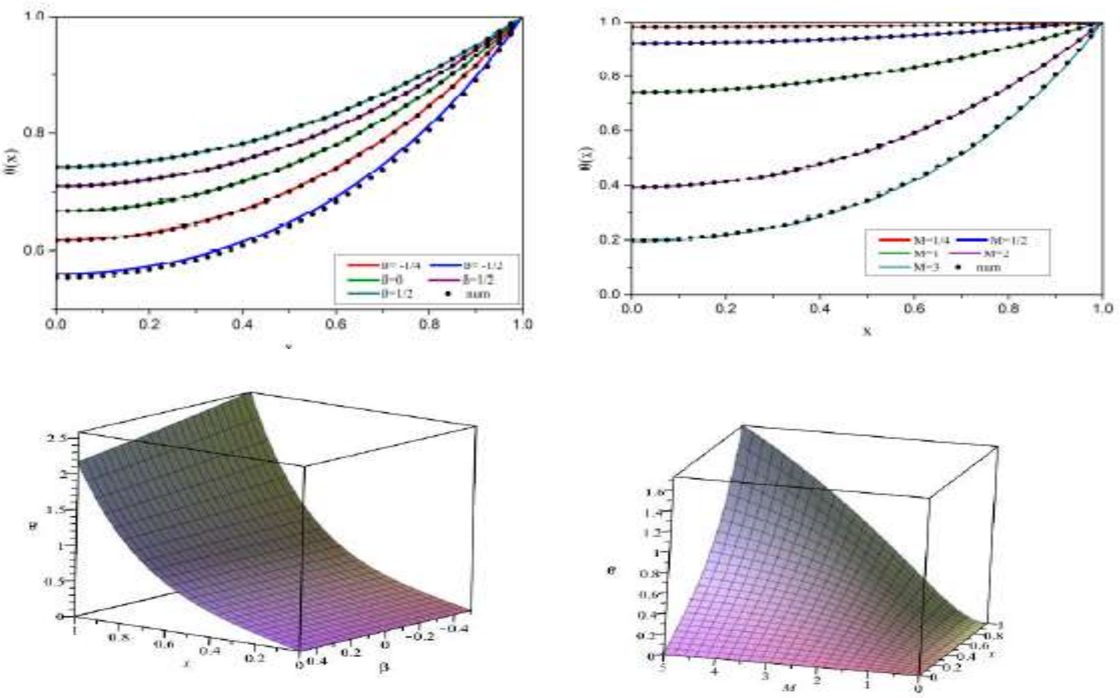


Figure shows comparative analysis LSM and CM

Table.1. The obtained Errors through both methods in comparison with numerical answer

X	ERROR IN LSM	ERROR IN CM
0	0.000138516	0.005797178
0.2	0.001134660	0.003903258
0.4	0.000027112	0.002292075
0.6	0.000940761	0.001526684
0.8	0.000249334	0.001562415
1	**0**	**0**

6.0 Conclusions

This paper investigated nonlinear heat transfer equations for a fin with power-law temperature-dependent thermal conductivity and thermal transfer coefficient, and this paper studied the method through strict analytical techniques for the resolution of a nonlinear differential equation.

In the Least Square Method and Collocation Method, the experimental function has been chosen to satisfy the limit conditions and continue the process of solution, the ultimate answer based on the test function is that by choosing more terms for the trial function the solution would be a little accidental but the process of solving it is important to note the test function.

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